

THE DYNKIN DIAGRAMS PACKAGE

VERSION 3.1

BEN MCKAY

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1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)

```
\usepackage{dynkin-diagrams}
```

Invoke it

The Dynkin diagram of (B_3) is $\text{dynkin}\{B\}\{3\}$.

The Dynkin diagram of B_3 is $\bullet \text{---} \bullet \Rightarrow \bullet$.

Inside a *TikZ* statement

```
\tikz \dynkin{B}{3};
```



Inside a *TikZ* environment

```
\begin{tikzpicture}
  \dynkin{B}{3}
\end{tikzpicture}
```



Indefinite rank Dynkin diagrams

```
\dynkin{B}{}
```

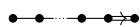


Table 1: The Dynkin diagrams of the reduced simple root systems
[2] pp. 265–290, plates I–IX

A_n		<code>\dynkin{A}{}{}</code>
C_n		<code>\dynkin{C}{}{}</code>
D_n		<code>\dynkin{D}{}{}</code>
E_6		<code>\dynkin{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
G_2		<code>\dynkin{G}{2}</code>

2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
```

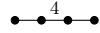
...or pass to the package

```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  mark=o,
  radius=.06cm]
{dynkin-diagrams}
```

3. COXETER DIAGRAMS

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```



gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \
\ (I_n=\dynkin[Coxeter,gonality=n]{I}{})
```

$$G_2 = \overset{n}{\bullet} \bullet, \quad I_n = \overset{n}{\bullet} \bullet$$

Table 2: The Coxeter diagrams of the simple reflection groups

A_n		<code>\dynkin[Coxeter]{A}{}</code>
B_n		<code>\dynkin[Coxeter]{B}{}</code>
C_n		<code>\dynkin[Coxeter]{C}{}</code>
E_6		<code>\dynkin[Coxeter]{E}{6}</code>
E_7		<code>\dynkin[Coxeter]{E}{7}</code>
E_8		<code>\dynkin[Coxeter]{E}{8}</code>
F_4		<code>\dynkin[Coxeter]{F}{4}</code>
G_2		<code>\dynkin[Coxeter,gonality=n]{G}{2}</code>
H_3		<code>\dynkin[Coxeter]{H}{3}</code>
H_4		<code>\dynkin[Coxeter]{H}{4}</code>
I_n		<code>\dynkin[Coxeter,gonality=n]{I}{}</code>

4. SATAKE DIAGRAMS

Satake diagrams use the standard name instead of a rank

$\backslash(A_{IIIb})=\backslash\text{dynkin}\{A\}\{IIIb\}\backslash$

$$A_{IIIb} = \begin{array}{c} \circ - \circ - \circ - \circ \\ | \quad | \quad | \quad | \\ \circ - \circ - \circ - \circ \end{array}$$












We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [9] p. 532–534

A_I		$\backslash\text{dynkin}\{A\}\{I\}$
A_{II}		$\backslash\text{dynkin}\{A\}\{II\}$
A_{IIIa}		$\backslash\text{dynkin}\{A\}\{IIIa\}$
A_{IIIb}		$\backslash\text{dynkin}\{A\}\{IIIb\}$
A_{IV}		$\backslash\text{dynkin}\{A\}\{IV\}$
B_I		$\backslash\text{dynkin}\{B\}\{I\}$
B_{II}		$\backslash\text{dynkin}\{B\}\{II\}$
C_I		$\backslash\text{dynkin}\{C\}\{I\}$
C_{IIa}		$\backslash\text{dynkin}\{C\}\{IIa\}$
C_{IIb}		$\backslash\text{dynkin}\{C\}\{IIb\}$
D_{Ia}		$\backslash\text{dynkin}\{D\}\{Ia\}$
D_{Ib}		$\backslash\text{dynkin}\{D\}\{Ib\}$
D_{Ic}		$\backslash\text{dynkin}\{D\}\{Ic\}$
D_{II}		$\backslash\text{dynkin}\{D\}\{II\}$
D_{IIIa}		$\backslash\text{dynkin}\{D\}\{IIIa\}$
D_{IIIb}		$\backslash\text{dynkin}\{D\}\{IIIb\}$
E_I		$\backslash\text{dynkin}\{E\}\{I\}$

continued ...

Table 3: ...continued

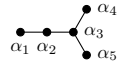
E_{II}		<code>\dynkin{E}{II}</code>
E_{III}		<code>\dynkin{E}{III}</code>
E_{IV}		<code>\dynkin{E}{IV}</code>
E_V		<code>\dynkin{E}{V}</code>
E_{VI}		<code>\dynkin{E}{VI}</code>
E_{VII}		<code>\dynkin{E}{VII}</code>
E_{VIII}		<code>\dynkin{E}{VIII}</code>
E_{IX}		<code>\dynkin{E}{IX}</code>
F_I		<code>\dynkin{F}{I}</code>
F_{II}		<code>\dynkin{F}{II}</code>
G_I		<code>\dynkin{G}{I}</code>

5. LABELS FOR THE ROOTS

Label the roots by root number

`\dynkin[label]{B}{3}`

Make a macro to assign labels to roots

`\dynkin[label,labelMacro/.code={\alpha_{#1}}]{D}{5}`

Label a single root

```

\begin{tikzpicture}
  \dynkin{B}{3}
  \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}

```



Use a text style

```
\begin{tikzpicture}
  \dynkin[text/.style={scale=1.2}]{B}{3};
  \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



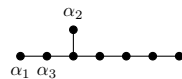
Access root labels via TikZ

```
\begin{tikzpicture}
  \dynkin{B}{3};
  \node[below] at (root 2) {\(\alpha_2\)};
\end{tikzpicture}
```



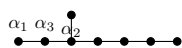
The labels have default locations

```
\begin{tikzpicture}
  \dynkin{E}{8};
  \dynkinLabelRoot{1}{\alpha_1}
  \dynkinLabelRoot{2}{\alpha_2}
  \dynkinLabelRoot{3}{\alpha_3}
\end{tikzpicture}
```



The starred form flips labels to alternate locations

```
\begin{tikzpicture}
  \dynkin{E}{8};
  \dynkinLabelRoot*{1}{\alpha_1}
  \dynkinLabelRoot*{2}{\alpha_2}
  \dynkinLabelRoot*{3}{\alpha_3}
\end{tikzpicture}
```



6. STYLE

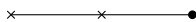
Colours

```
\dynkin[edge/.style={blue!50,thick},*/.style=blue!50!red]{F}{4}
```



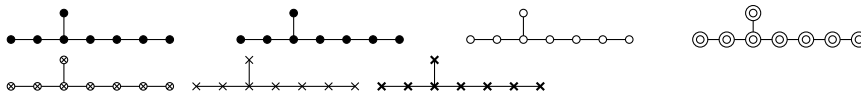
Edge lengths

```
\dynkin[edgeLength=1.2,parabolic=3]{A}{3}
```



Root marks

```
\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=O]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}
```



At the moment, you can only use:

- * solid dot
- o hollow circle
- O double hollow circle
- t tensor root
- x crossed root
- X thickly crossed root

Mark styles

```
\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}
```



Sizes of root marks

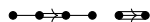
```
\dynkin[radius=.08cm,parabolic=3]{A}{3}
```



7. SUPPRESS OR REVERSE ARROWS

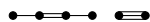
Some diagrams have double or triple edges

```
\dynkin{F}{4}
\dynkin{G}{2}
```



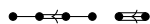
Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```



Reverse arrows

```
\dynkin[reverseArrows]{F}{4}
\dynkin[reverseArrows]{G}{2}
```



8. DRAWING ON TOP OF A DYNKIN DIAGRAM

TikZ can access the roots themselves

```
\begin{tikzpicture}
  \dynkin{A}{4};
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
```

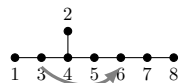


```
\end{tikzpicture}
```



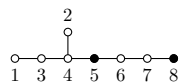
Draw curves between the roots

```
\begin{tikzpicture}
  \dynkin[label]{E}{8}
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{tikzpicture}
```



Change marks

```
\begin{tikzpicture}
  \dynkin[mark=o,label]{E}{8};
  \dynkinRootMark{*}{5}
  \dynkinRootMark{*}{8}
\end{tikzpicture}
```

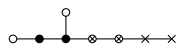


9. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, \dots , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

Table 4: Classical Lie superalgebras [7]. We need a slightly larger radius parameter to distinguish the tensor product symbols from the solid dots.

A_{mn}		<code>\dynkin{A}{ooo.oto.oo}</code>
B_{mn}		<code>\dynkin{B}{ooo.oto.oo}</code>
B_{0n}		<code>\dynkin{B}{ooo.ooo.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{ooo.oto.oooo}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affineMark=t]{G}{2}</code>

Table 5: Classical Lie superalgebras [7]. Here we see the problem with using the default radius parameter, which is too small for tensor product symbols.

A_{mn}		<code>\dynkin{A}{ooo.oto.oo}</code>
B_{mn}		<code>\dynkin{B}{ooo.oto.oo}</code>
B_{0n}		<code>\dynkin{B}{ooo.ooo.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{ooo.oto.oooo}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affineMark=t]{G}{2}</code>

10. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots, $\bullet \cdots \bullet$ indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

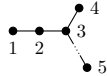
`\dynkin{D}{o.o*.*.t.to.t}`



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

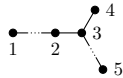
Indefinite edge option

```
\dynkin[makeIndefiniteEdge={3-5},label]{D}{5}
```



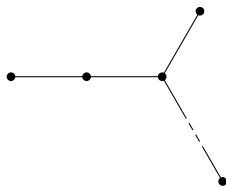
Give a list of edges to become indefinite

```
\dynkin[makeIndefiniteEdge/.list={1-2,3-5},label]{D}{5}
```



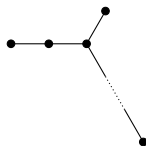
Indefinite edge style

```
\dynkin[indefiniteEdge/.style={draw=black,fill=white,thin,densely
dashed},%
edgeLength=1cm,%
makeIndefiniteEdge={3-5}]
{D}{5}
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edgeLength = .5cm,%
indefiniteEdgeRatio=3,%
makeIndefiniteEdge={3-5}]
{D}{5}
```



11. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\backslash\mathrm{dynkin}[\mathrm{parabolic}=3]\{\mathrm{A}\}\{3\}$.

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\times\text{---}\bullet$.

Table 6: The Hermitian symmetric spaces

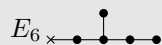
A_n	$\bullet\text{---}\bullet\text{---}\times\text{---}\bullet\text{---}\bullet$	$\backslash\mathrm{dynkin}\{\mathrm{A}\}\{\ast\ast.\ast\times.\ast\ast\}$	Grassmannian of k -planes in \mathbb{C}^{n+1}
B_n	$\times\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\rightarrow\bullet$	$\backslash\mathrm{dynkin}[\mathrm{parabolic}=1]\{\mathrm{B}\}\{\}$	$(2n - 1)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n+1}
C_n	$\bullet\text{---}\bullet\text{---}\bullet\text{---}\leftarrow\times$	$\backslash\mathrm{dynkin}[\mathrm{parabolic}=16]\{\mathrm{C}\}\{\}$	space of Lagrangian n -planes in \mathbb{C}^{2n}
D_n	$\times\text{---}\bullet\text{---}\bullet\text{---}\begin{array}{c} \bullet \\ \diagup \\ \bullet \end{array}$	$\backslash\mathrm{dynkin}[\mathrm{parabolic}=1]\{\mathrm{D}\}\{\}$	$(2n - 2)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n}
D_n	$\bullet\text{---}\bullet\text{---}\bullet\text{---}\begin{array}{c} \bullet \\ \diagup \\ \bullet \\ \diagdown \\ \times \end{array}$	$\backslash\mathrm{dynkin}[\mathrm{parabolic}=32]\{\mathrm{D}\}\{\}$	one component of the variety of maximal dimension null subspaces of \mathbb{C}^{2n}

Table 6: continued ...

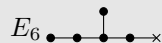
Table 6: ...continued


 $\backslash\mathrm{dynkin}[\mathrm{parabolic}=16]\{\mathrm{D}\}\{\}$

the other component


 $\backslash\mathrm{dynkin}[\mathrm{parabolic}=1]\{\mathrm{E}\}\{6\}$

complexified octave projective plane


 $\backslash\mathrm{dynkin}[\mathrm{parabolic}=32]\{\mathrm{E}\}\{6\}$

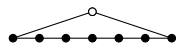
its dual plane


 $\backslash\mathrm{dynkin}[\mathrm{parabolic}=64]\{\mathrm{E}\}\{7\}$

the space of null octave 3-planes in octave 6-space

12. EXTENDED DYNKIN DIAGRAMS

Extended Dynkin diagrams

 $\backslash\mathrm{dynkin}[\mathrm{extended}]\{\mathrm{A}\}\{7\}$


The extended Dynkin diagrams are also described in the notation of Kac [11] p. 55 as affine untwisted Dynkin diagrams: we extend $\backslash\mathrm{dynkin}\{\mathrm{A}\}\{7\}$ to become $\backslash\mathrm{dynkin}\{\mathrm{A}\}[1]\{7\}$:

Extended Dynkin diagrams

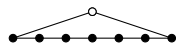
 $\backslash\mathrm{dynkin}\{\mathrm{A}\}[1]\{7\}$


Table 7: The Dynkin diagrams of the extended simple root systems

A_1		<code>\dynkin[extended]{A}{1}</code>
A_n		<code>\dynkin[extended]{A}{}</code>
B_n		<code>\dynkin[extended]{B}{}</code>
C_n		<code>\dynkin[extended]{C}{}</code>
D_n		<code>\dynkin[extended]{D}{}</code>
E_6		<code>\dynkin[extended]{E}{6}</code>
E_7		<code>\dynkin[extended]{E}{7}</code>
E_8		<code>\dynkin[extended]{E}{8}</code>
F_4		<code>\dynkin[extended]{F}{4}</code>
G_2		<code>\dynkin[extended]{G}{2}</code>

13. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [11] p. 55:

Affine Dynkin diagrams

$\backslash(A^{(1)})_7=\backslash\mathrm{dynkin}\{A\}[1]\{7\}, \backslash$
 $E^{(2)}_6=\backslash\mathrm{dynkin}\{E\}[2]\{6\}, \backslash$
 $D^{(3)}_4=\backslash\mathrm{dynkin}\{D\}[3]\{4\}\backslash$

$A_7^{(1)} =$ $, E_6^{(2)} =$ $, D_4^{(3)} =$

Table 8: The affine Dynkin diagrams

A_1^1		<code>\dynkin{A}[1]{1}</code>
A_n^1		<code>\dynkin{A}[1]{}</code>
B_n^1		<code>\dynkin{B}[1]{}</code>
C_n^1		<code>\dynkin{C}[1]{}</code>
D_n^1		<code>\dynkin{D}[1]{}</code>

continued ...

Table 8: ...continued

E_6^1		<code>\dynkin{E}[1]{6}</code>
E_7^1		<code>\dynkin{E}[1]{7}</code>
E_8^1		<code>\dynkin{E}[1]{8}</code>
F_4^1		<code>\dynkin{F}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
A_2^2		<code>\dynkin{A}[2]{2}</code>
A_{even}^2		<code>\dynkin{A}[2]{even}</code>
A_{odd}^2		<code>\dynkin{A}[2]{odd}</code>
D_n^2		<code>\dynkin{D}[2]{}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>

Table 9: Some more affine Dynkin diagrams

A_4^2		<code>\dynkin{A}[2]{4}</code>
A_5^2		<code>\dynkin{A}[2]{5}</code>
A_6^2		<code>\dynkin{A}[2]{6}</code>
A_7^2		<code>\dynkin{A}[2]{7}</code>
A_8^2		<code>\dynkin{A}[2]{8}</code>
D_3^2		<code>\dynkin{D}[2]{3}</code>
D_4^2		<code>\dynkin{D}[2]{4}</code>
D_5^2		<code>\dynkin{D}[2]{5}</code>
D_6^2		<code>\dynkin{D}[2]{6}</code>
D_7^2		<code>\dynkin{D}[2]{7}</code>
D_8^2		<code>\dynkin{D}[2]{8}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>

continued ...

Table 9: ...continued

E_6^2		<code>\dynkin{E}[2]{6}</code>
---------	--	-------------------------------

14. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

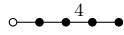
`\dynkin[extended,Coxeter]{F}{4}`

Table 10: The extended (affine) Coxeter diagrams

A_n		<code>\dynkin[extended,Coxeter]{A}{}{}</code>
B_n		<code>\dynkin[extended,Coxeter]{B}{}{}</code>
C_n		<code>\dynkin[extended,Coxeter]{C}{}{}</code>
D_n		<code>\dynkin[extended,Coxeter]{D}{}{}</code>
E_6		<code>\dynkin[extended,Coxeter]{E}{6}</code>
E_7		<code>\dynkin[extended,Coxeter]{E}{7}</code>
E_8		<code>\dynkin[extended,Coxeter]{E}{8}</code>
F_4		<code>\dynkin[extended,Coxeter]{F}{4}</code>
G_2		<code>\dynkin[extended,Coxeter]{G}{2}</code>
H_3		<code>\dynkin[extended,Coxeter]{H}{3}</code>
H_4		<code>\dynkin[extended,Coxeter]{H}{4}</code>
I_1		<code>\dynkin[extended,Coxeter]{I}{1}</code>

15. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [11].

Kac style

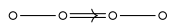
`\dynkin[Kac]{F}{4}`

Table 11: The Dynkin diagrams of the extended simple root systems in Kac style. At the moment, it only works on a white background.

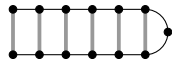
A_1		<code>\dynkin[extended]{A}{1}</code>
A_n		<code>\dynkin[extended]{A}{}</code>
B_n		<code>\dynkin[extended]{B}{}</code>
C_n		<code>\dynkin[extended]{C}{}</code>
D_n		<code>\dynkin[extended]{D}{}</code>
E_6		<code>\dynkin[extended]{E}{6}</code>
E_7		<code>\dynkin[extended]{E}{7}</code>
E_8		<code>\dynkin[extended]{E}{8}</code>
F_4		<code>\dynkin[extended]{F}{4}</code>
G_2		<code>\dynkin[extended]{G}{2}</code>

16. FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

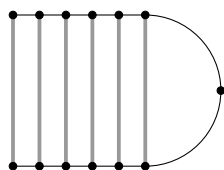
Folding

`\dynkin[fold]{A}{13}`



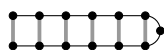
Big fold radius

`\dynkin[fold,foldradius=1cm]{A}{13}`



Small fold radius

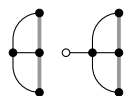
```
\dynkin[fold,foldradius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

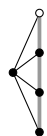
3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



4-ply

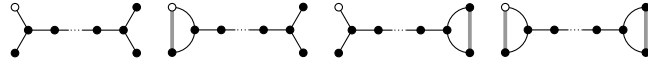
```
\dynkin[ply=4]{D}[1]{4}
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{ } \
\dynkin[foldleft]{D}[1]{ } \
\dynkin[foldright]{D}[1]{ } \
\dynkin[fold]{D}[1]{ }
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
\begin{tikzpicture}
  \dynkin[ply=4]{D}[1]{****.*****.****}%
\end{tikzpicture} \
\begin{tikzpicture}
  \dynkin[ply=4]{D}[1]{****.*****.****}%
  \dynkinFold[bend right=65]{1}{13}%
  \dynkinFold[bend right=65]{0}{14}%
\end{tikzpicture} \
\begin{tikzpicture}
  \dynkin[ply=4]{D}[1]{****.*****.****}%
  \dynkinFold{0}{1}%
  \dynkinFold{1}{13}%
  \dynkinFold{13}{14}%
\end{tikzpicture}
```

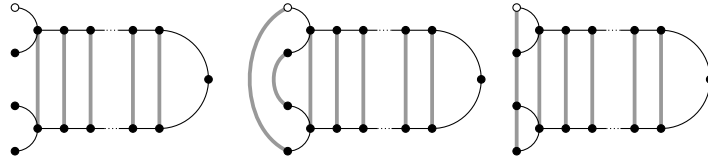


Table 12: Some foldings of Dynkin diagrams

A_3		<code>\dynkin[fold]{A}[0]{3}</code>
C_2		<code>\dynkin{C}[0]{2}</code>
$A_{2\ell-1}$		<code>\dynkin[fold]{A}{**.*.....**}</code>
C_ℓ		<code>\dynkin{C}{}</code>

continued ...

Table 12: ...continued

B_3		<code>\dynkin[fold]{B}[0]{3}</code>
G_2		<code>\dynkin[reverseArrows]{G}[0]{2}</code>
D_4		<code>\dynkin[ply=3]{D}{4}</code>
G_2		<code>\dynkin{G}{2}</code>
$D_{\ell+1}$		<code>\dynkin[fold]{D}{}</code>
B_ℓ		<code>\dynkin{B}{}</code>
E_6		<code>\dynkin[fold]{E}[0]{6}</code>
F_4		<code>\dynkin[reverseArrows]{F}[0]{4}</code>
A_3^1		<code>\dynkin[ply=4]{A}[1]{3}</code>
A_1^1		<code>\dynkin{A}[1]{1}</code>
$A_{2\ell-1}^1$		<code>\dynkin[fold]{A}[1]{**.*.....**}</code>
C_ℓ^1		<code>\dynkin{C}[1]{}</code>
B_3^1		<code>\dynkin[ply=3]{B}[1]{3}</code>
A_2^2		<code>\dynkin{A}[2]{2}</code>
B_3^1		<code>\dynkin[ply=2]{B}[1]{3}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
B_ℓ^1		<code>\dynkin[fold]{B}[1]{}</code>
D_ℓ^2		<code>\dynkin{D}[2]{}</code>

continued ...

Table 12: ...continued

D_4^1		<code>\dynkin[ply=3]{D}[1]{4}</code>
B_3^1		<code>\dynkin{B}[1]{3}</code>
D_4^1		<code>\dynkin[ply=3]{D}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold]{D}[1]{}</code>
D_ℓ^2		<code>\dynkin{D}[2]{}</code>
$D_{\ell+1}^1$		<code>\dynkin[foldright]{D}[1]{}</code>
B_ℓ^1		<code>\dynkin{B}[1]{}</code>
$D_{2\ell}^1$		<pre> \begin{tikzpicture} \dynkin[ply=4]{D}[1]{****.*****.*****} \dynkinFold{0}{1} \dynkinFold{1}{13} \dynkinFold{13}{14} \end{tikzpicture> </pre>
A_{odd}^2		<code>\dynkin{A}[2]{odd}</code>

continued ...

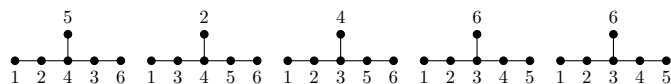
Table 12: ...continued

$D_{2\ell}^1$		<pre>\begin{tikzpicture} \dynkin[ply=4]{D}[1]{****.*****.*****} \dynkinFold[bend right=65]{1}{13} \dynkinFold[bend right=65]{0}{14} \end{tikzpicture}</pre>
A_{even}^2		<pre>\dynkin{A}[2]{even}</pre>
E_6^1		<pre>\dynkin[fold]{E}[1]{6}</pre>
F_4^1		<pre>\dynkin[reverseArrows]{F}[1]{4}</pre>
E_6^1		<pre>\dynkin[ply=3]{E}[1]{6}</pre>
D_4^3		<pre>\dynkin{D}[3]{4}</pre>
E_7^1		<pre>\dynkin[fold]{E}[1]{7}</pre>
E_6^2		<pre>\dynkin{E}[2]{6}</pre>
F_4^1		<pre>\dynkin[fold]{F}[1]{4}</pre>
G_2^1		<pre>\dynkin{G}[1]{2}</pre>
A_{odd}^2		<pre>\dynkin[odd, fold]{A}[2]{****.***}</pre>
A_{even}^2		<pre>\dynkin{A}[2]{even}</pre>
D_3^2		<pre>\dynkin[fold]{D}[2]{3}</pre>
A_2^2		<pre>\dynkin{A}[2]{2}</pre>

17. ROOT ORDERING

Root ordering

```
\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}
```



Default is Bourbaki.

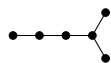
	Adams	Bourbaki	Carter	Dynkin	Kac
E_6					
E_7					
E_8					
F_4					
G_2					

18. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:

Connect diagrams

```

\begin{tikzpicture}
  \dynkin[name=upper]{A}{3}
  \node (current) at ($(\text{upper root 1})+(0,-.3\text{cm})$) {};
  \dynkin[at=(current),name=lower]{A}{3}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/foldStyle]
        ($(\text{upper root } \i)$) -- ($(\text{lower
        root } \i)$);%
    }%
  \end{scope}
\end{tikzpicture}

```

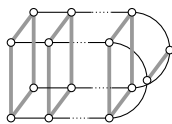


The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [1].

```

\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]{A}{IIb}
  \node (a) at (.3,.4){};
  \dynkin[name=2,at=(a)]{A}{IIb}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,7}%
    {%
      \draw[/Dynkin diagram/foldStyle]
        ($(\text{1 root } \i)$)
        --
        ($(\text{2 root } \i)$);%
    }%
  \end{scope}
\end{tikzpicture}

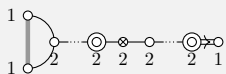
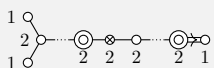
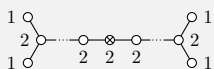
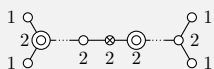
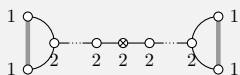
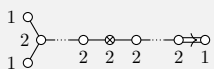
```

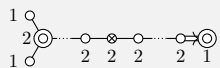
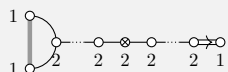
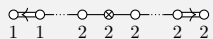
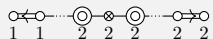
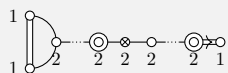
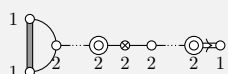


```

\pgfkeys{/Dynkin diagram/edgeLength=.75cm,/Dynkin
diagram/edge/.style={draw=white,double=black,very thick},
}

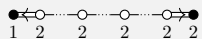
```


$\backslash\text{dynkin}[\text{ply}=2,\text{label}]\{\text{B}\}[1]\{\text{oo.0to.0o}\}$

 $\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{oo.0to.0o}\}$

 $\backslash\text{dynkin}[\text{label}]\{\text{D}\}[1]\{\text{oo.oto.ooo}\}$

 $\backslash\text{dynkin}[\text{label}]\{\text{D}\}[1]\{\text{o0.ot0.ooo}\}$

 $\backslash\text{dynkin}[\text{label},\text{fold}]\{\text{D}\}[1]\{\text{oo.oto.ooo}\}$

 $\mathfrak{sl}(2m+1|2n)^2$
 $\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{oo.oto.oo}\}$


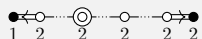
$\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{o}0.\text{o}t\text{o}.\text{o}0\}$

 $\backslash\text{dynkin}[\text{label},\text{fold}]\{\text{B}\}[1]\{\text{o}0.\text{o}t\text{o}.\text{o}0\}$

 $\mathfrak{sl}(2m+1|2n+1)^2$
 $\backslash\text{dynkin}[\text{label}]\{\text{D}\}[2]\{\text{o}.\text{o}t\text{o}.\text{o}0\}$

 $\backslash\text{dynkin}[\text{label}]\{\text{D}\}[2]\{\text{o}.\text{o}t\text{o}.\text{o}0\}$

 $\mathfrak{sl}(2|2n+1)^{(2)}$
 $\backslash\text{dynkin}[\text{ply}=2,\text{label},\text{doubleEdges}]\{\text{B}\}[1]\{\text{o}0.\text{o}t\text{o}.\text{o}0\}$

 $\backslash\text{dynkin}[\text{ply}=2,\text{label},\text{doubleFold}]\{\text{B}\}[1]\{\text{o}0.\text{o}t\text{o}.\text{o}0\}$


$\mathfrak{osp}(2|2n)^{(2)}$

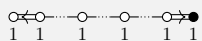
```
\dynkin[label,labelMacro/.code=\lablIt{#1},
  affineMark=*]
{D}[2]{o.o.o.o*}
```



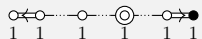
```
\dynkin[label,labelMacro/.code=\lablIt{#1},
  affineMark=*]
{D}[2]{o.O.o.o*}
```


 $\mathfrak{sl}(1|2n+1)^4$

```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.o.o*}
```



```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.O.o*}
```



A^1

```

\begin{tikzpicture}
  \dynkin[name=upper]{A}{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin
current),name=lower]{A}{oo.t.oo}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,5}{
      \draw[/Dynkin diagram/foldStyle]
        ($(\upper root \i)$) --
        ($(\lower root \i)$);
    }
  \end{scope}
\end{tikzpicture}

```



```

\dynkin[fold]{A}[1]{oo.t.ooooo.t.oo}

```



```

\dynkin[fold,affineMark=t]{A}[1]{oo.o.ootoo.o.oo}

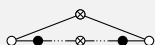
```



```

\dynkin[affineMark=t]{A}[1]{o*.t.*o}

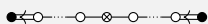
```

 B^1

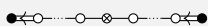
```

\dynkin[affineMark=*]{A}[2]{o.oto.o*}

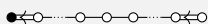
```



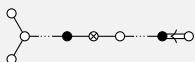
```
\dynkin[affineMark=*]{A}[2]{o.oto.o*}
```



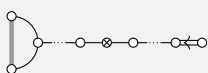
```
\dynkin[affineMark=*]{A}[2]{o.ooo.o}
```



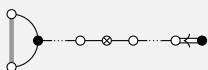
```
\dynkin[odd]{A}[2]{oo.*to.*o}
```



```
\dynkin[odd,fold]{A}[2]{oo.oto.oo}
```



```
\dynkin[odd,fold]{A}[2]{o*.oto.o*}
```


 D^1

```
\dynkin{D}{otoo}
```



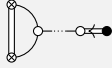
```
\dynkin{D}{ot*o}
```



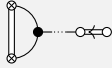
```
\dynkin[fold]{D}{otoo}
```


 C^1

```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{to.o*}
```



```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{t*.oo}
```


 F^1

```
\begin{tikzpicture}%
  \dynkin{A}{oto*}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```



```
\begin{tikzpicture}%
  \dynkin{A}{*too}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```

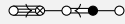


G^1

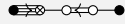
```
\begin{tikzpicture}%
  \dynkin{A}{ot*oo}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



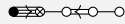
```
\begin{tikzpicture}%
  \dynkin{A}{oto*o}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



```
\begin{tikzpicture}%
  \dynkin{A}{*too*}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



```
\begin{tikzpicture}%
  \dynkin{A}{*tooo}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



20. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram,

`<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type ⁽¹⁾
- 2 affine twisted root system of type ⁽²⁾
- 3 affine twisted root system of type ⁽³⁾

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

21. OPTIONS

```

text/.style = {TikZ style data},
default : scale=.7
           Style for any labels on the roots.
name = {string},
default : anonymous
           A name for the Dynkin diagram, with anonymous treated as a
           blank; see section 18.
parabolic = {integer},
default : 0
           A parabolic subgroup with specified integer, where the integer
           is computed as  $n = \sum 2^{i-1} a_i$ ,  $a_i = 0$  or  $1$ , to say that root  $i$  is
           crossed, i.e. a noncompact root.
radius = {number}cm,
default : .05cm
           size of the dots and of the crosses in the Dynkin diagram
edgeLength = {number}cm,
default : .35cm
           distance between nodes in the Dynkin diagram
edge/.style = TikZ style data,
default : thin
           style of edges in the Dynkin diagram
mark = {o,0,t,x,X,*},
default : *
           default root mark
affineMark = o,0,t,x,X,*,
default : *
           default root mark for root zero in an affine Dynkin diagram
label = true or false,
default : false
           whether to label the roots according to the current labelling scheme.
labelMacro = {1-parameter TEX macro},
default : #1
           the current labelling scheme.
makeIndefiniteEdge = {edge pair  $i$ - $j$  or list of such},
default : {}

```

continued ...

Table 14: ...continued

edge pair or list of edge pairs to treat as having indefinitely many roots on them.

`indefiniteEdgeRatio = <float>`,
 default : 1.6
 ratio of indefinite edge lengths to other edge lengths.

`indefiniteEdge/.style = <TikZ style data>`,
 default : `draw=black,fill=white,thin,densely dotted`
 style of the dotted or dashed middle third of each indefinite edge.

`arrows = <true or false>`,
 default : `true`
 whether to draw the arrows that arise along the edges.

`reverseArrows = <true or false>`,
 default : `true`
 whether to reverse the direction of the arrows that arise along the edges.

`fold = <true or false>`,
 default : `true`
 whether, when drawing Dynkin diagrams, to draw them 2-ply.

`ply = <0,1,2,3,4>`,
 default : 0
 how many roots get folded together, at most.

`foldleft = <true or false>`,
 default : `true`
 whether to fold the roots on the left side of a Dynkin diagram.

`foldright = <true or false>`,
 default : `true`
 whether to fold the roots on the right side of a Dynkin diagram.

`foldradius = <length>`,
 default : .3cm
 the radius of circular arcs used in curved edges of folded Dynkin diagrams.

`foldStyle = <TikZ style data>`,
 default : `draw=black!40,fill=none,line width=radius`
 when drawing folded diagrams, style for the fold indicators.

`*/.style = <TikZ style data>`,
 default : `draw=black,fill=black`
 style for roots like \bullet

`o/.style = <TikZ style data>`,
 default : `draw=black,fill=black`
 style for roots like \circ

`O/.style = <TikZ style data>`,
 default : `draw=black,fill=black`
 style for roots like \odot

`t/.style = <TikZ style data>`,
 default : `draw=black,fill=black`
 style for roots like \otimes

continued ...

Table 14: ...continued

```

x/.style = <TikZ style data>,
default : draw=black
           style for roots like  $\times$ 
X/.style = <TikZ style data>,
default : draw=black,thick
           style for roots like  $\times$ 
leftFold/.style = <TikZ style data>,
default :
           style to override the fold style when folding roots together on the
           left half of a Dynkin diagram
rightFold/.style = <TikZ style data>,
default :
           style to override the fold style when folding roots together on the
           right half of a Dynkin diagram
doubleEdges = <>,
default : not set
           set to override the fold style when folding roots together in a
           Dynkin diagram, so that the foldings are indicated with double
           edges (like those of an  $F_4$  Dynkin diagram without arrows).
doubleFold = <>,
default : not set
           set to override the fold style when folding roots together in a
           Dynkin diagram, so that the foldings are indicated with double
           edges (like those of an  $F_4$  Dynkin diagram without arrows), but
           filled in solidly.
doubleLeft = <>,
default : not set
           set to override the fold style when folding roots together at the
           left side of a Dynkin diagram, so that the foldings are indicated
           with double edges (like those of an  $F_4$  Dynkin diagram without
           arrows).
doubleFoldLeft = <>,
default : not set
           set to override the fold style when folding roots together at the
           left side of a Dynkin diagram, so that the foldings are indicated
           with double edges (like those of an  $F_4$  Dynkin diagram without
           arrows), but filled in solidly.
doubleRight = <>,
default : not set
           set to override the fold style when folding roots together at the
           right side of a Dynkin diagram, so that the foldings are indicated
           with double edges (like those of an  $F_4$  Dynkin diagram without
           arrows).
doubleFoldRight = <>,
default : not set

```

continued ...

Table 14: ...continued

set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`Coxeter` = `<true or false>`,

default : `false`

whether to draw a Coxeter diagram, rather than a Dynkin diagram.

`ordering` = `<Adams, Bourbaki, Carter, Dynkin, Kac>`,

default : `Bourbaki`

which ordering of the roots to use in exceptional root systems as in section 17.

All other options are passed to TikZ.

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